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GRAPH OPTIMIZATION ALGORITHM – A CASE STUDY

MILAN MARTIC
JELENA DRAGICEVIC
Faculty of Organizational Science, University of Belgrade, Serbia and Montenegro

DARKO KRULJ
Trizon Group, Belgrade, Serbia and Montenegro

Abstract
The paper shows the problem solving task of finding the graphs shortest paths by using the Dijkstra's algorithm, matched with contemporary methods, instruments and techniques developed within geographical information systems (GIS). The concept of digital cartography is defined as well as the advantages of digital maps usage. The paper gives an idea of a layout of future digital maps and suggests various possibilities offered by GIS with the purpose of spatial positioning of moving objects. The paper is complemented with presentation of practical implementation of GIS in the process of solving the problem of determining the graphs shortest paths, giving the several suitable examples, which helps us to present, in very simple way, effective connection between an algorithm for graph optimization and up-to-date technological solutions.

***Keywords**: graph optimization, shortest path, GIS*

1. INTRODUCTION

Since the age of Eratosthenes, who first drew the borders of the world known at the time, a man has been using maps "to see" the world too big and complicated to be seen with naked eye. The map helps us show and realize relationships in space in the same way as written or spoken words help us understand our companion. We must point out that the map doesn't represent a diminished, ideal picture of the world but an abstract picture of something what is closer to us for understanding than the whole space of our planet. This exact character, with the help of modern technology (by using PC-s, Internet, and different cartographic software's), has given each of us an opportunity to create one's own picture of the world, a picture of good quality, that has been a privilege only of the eminent cartographic experts up to now.

A map, [5] according to definition, is general and mathematically developed picture of the surface of the world or some of its part.

There are two important functions of the map:

- A map is used as media for storing information necessary for mankind
- A map used as a picture of the world that helps us understand formulas and relationships in space as well as complexity of the surroundings we're living in

Cartography, according to definition [5], is a branch of work that provides making and usage of maps, as a special kind of representing space position.

Usage of computer technology in cartography is known as digital cartography. The usage of computers has had an influence on special kind of changes or addition to functions of analogue maps. Some of these changes are:

- A digital data base which substitutes a printed map, as a media for storing geographical information
- A cartographic visualization in many different medias satisfies needs, for which printed maps had been used before.

In order to define the meaning of "digital map" it is useful to start from the definition of "ordinary", namely analogue map as graphic representation of geographic space. Digital map could be defined as any cartographic visualization in digital format, which can be shown on a screen of PC, or can be printed.

Taking into account the complexity, some authors make distinction between digital and electronic map. Representation in vector and/or raster format based on bearers suitable for computer processing is called a digital map. That kind of map has software and all the attributes necessary for showing on a computer screen, or drawing with plotter including the complete signalization, names and map description.

Interactive cartographic structure used for searching and showing information, which consists of one or more maps represented in raster or vector format and date base with descriptive information about some objects is cold an electronic map. This kind of map also contains software for searching and showing maps and descriptive data on PC screen or working station. Beside maps and text electronic map contains the sound and moving or not moving pictures.

2. DIGITAL CARTOGRAPHY

The Usage of computers and other electronic devises (e.g. Global Position System-GPS) and satellites has brought several definite solutions, which have advanced the process of making maps. GPS according to [4] is system developed to collect, first of all, position coordinates of any spot on the earth's surface.

Maybe, the most important advantage of using new technologies is less time that is needed to be spending for drawing maps. Traditionally, that process took to much time, and it was complex, and by the time when the "new" one. Constant collecting of data (e.g. by satellite) and their entering in digital form provides remarkably improving updates of the map content. Usage of high-resolution printers and plotters provides faster drawing, and on the other hand, it provides the whole process of drawing maps much cheaper.

Let us point out some more advantages of digital maps:

- Easy finding (e.g. automatic positioning) space dimension data on the map (if the visualization is connected with data base e.g. visualization of some settlement, address or some object- hospitals, gas stations, etc.)
- Easy making and usage of thematic maps with the possibility of choosing layers on digital map with certain contents (e.g. data about temperature, flows, population structure, population numerousness, gross national income by an inhabitant)
- The possibility of connecting and diminishing map parts (so called zooming)
- Possibility of moving maps and representing according to user's wish (so called pan)
- Usage of function of hipper connections and integrated multimedia contents related to data on the map (picture, sound, video, animation)
- Digital map on Internet, under the condition of regular updating, is the most used kind of cartographic visualization at the time
- Digital map on Internet, connected to data base, gives us an opportunity for synchronic and asynchrony collaboration projects (e.g. in space planning)

Main disadvantage and limitations of analogue maps are:

- Disadvantage of simplicity of manipulating with map, especially on the field (bending, folding over, drawing, measuring on map)
- Limited size of representation (the size of screen)
- Screen resolution and colors limit the representation of details because analogue map (on paper) has bigger resolution and probably a possibility for showing more details and information from digital map in the same proportion.

According to [1,7], the geographical information systems (GIS), is made of integrated group of data and procedures which provide gathering, modeling, manipulation, seeing, analyzing and presentation geographical data. Geographical information systems have been made for many different purposes, and they use different types of digital maps. Most frequently, GIS has been used with the purpose of spatial positioning of moving objects (ships, cars, aero planes, etc.), also for different investment analyses (road making, choosing the optimal location, etc.). More about GIS systems based on GPS can be find in [4,6]

Digital cartography, including GIS, gives numerous possibilities to geography. Some of them are:

- More modern space analyses, synthesis and modeling with GIS
- Faster and more simple (technically) drawing thematic maps
- Better quality of drawn thematic maps
- For geographic education (on every level)
- Drawing digital maps as a part of process for realizing space relationships
- Easy understanding of abstract proposition about geospace using new technologies of geovisualisation- multimedia, interactions, animations and 3D modeling.

One thing can't be denied; cartography is in very important changing process. But before discussing about, is, what we now call map, only static element of the map in originally context, or it still gives an opportunity for interactive cartographic approach, it is important to remark what all that changes mean for comprehension of space appearances, structures, and appearances in context of geography. That is to say, approximately 80% of all digital data has the geospace reference, e.g. geographic coordinates, addresses and postal codes.

At the end it's important to point out that many authors claim that information revolution consequences in cartography haven't realized all the possibilities yet, and that the digital cartography of the (close) future will be remarkably differ from the one that exists today.

This paper is based on transportation GIS. It includes the optimization algorithms on graphs, which will be explained in continue.

3. ALGORITHM

Let G = (N, L) be a graph, and suppose that lengths $c_{i,j}$ (i,j) ∈ L. are joint to the trees.

The aim is to find the shortest path via set nodes s and t. the path between set nodes s and t defines as line of trees, firs of them starts from node s, and any other following tree of the line starts at the node in which ends preliminary one, and the last tree ends in node t: $[(s, i_1), (i_1, i_2), \ldots, (i_{k-1}, i_k), (i_k, t)]$.

Alternatively, path could be defined as line of nodes if through them go: $(s,\ i_1,\ i_2, \ldots, i_{k-1},\ i_k,\ t)$.

Length of a path is sum of the weights of its constituent edges. The distance from s to t is the length of the shortest (minimum) length path.

Only basic paths through the tree are interesting. Basic path is path that goes via every node of the graph maximum one time.

DIJKSTRA'S ALGORITHM

This algorithm [3] is considered as the most efficient algorithm for solving the shortest path problem between two vertices, if condition that all edge weights is non-negative is satisfied, the aim>0 always for (i,j) ∈ L.

An idea of the algorithm and the proof that is built in for are based on optimization principle that can be simply described by claim: a sub path of an optimal path is also optimal.

Algorithm builds up the shortest path threes, edge-by-edge, starting from the first node and on each step adding one new edge, corresponding to construction of the optimization principle.

In the first iteration there has to be find the node closest to the root node, in the second iteration there has to be find the next (second) closest node to the root one. It can be reach either directly from a tree of shortest path beginning with root edge, or from the node that was marked as the closest to root, in preliminary iteration. The shortest path, from root to two nodes of tree, is labeled that way. Further the tree is built up edge by edge, solving the shortest path problem. The algorithm has a general rule that has to be followed: the shortest path to a node can be build directly from a root node or from an other node that was preliminary labeled as shortest one. At the end of the algorithm we obtain the weight of the minimal path from root to the last node, together with arrows which show this minimal path traced backward.

In purpose of implementation the optimization principle Dijkstra's algorithm uses concept of labeling the nodes. Label or mark d(j) of node j could be "temporarily labeled " which is assigned as d-(j), or node could be "permanently labeled" which is assigned as d+(j). To be "permanently labeled" means that value d(j) is equal to the length of a shortest path from the root node to the node j.

The algorithm and the example have been taken from [6].

Dijkstra's algorithm has six steps that will be represented.

1° Initialization

At the beginning of algorithm nodes are assigned this way:

The root node is assigned as "permanently labeled" d+(s) = 0;

Any other node is assigned as "temporarily labeled"

$$\mathbf{d\text{ - }(s)} = \infty\mathbf{,\ j} \in \mathbf{N\ /}\ \{\mathbf{s}\}$$

- Suppose that $i = s$ i

2° Define group A; that represents nodes which follow the labeled node and which don't have permanent label

$$Ai = \{ j \,/\, j \in \Gamma(i) \wedge d(j) = d-(j) \}$$

3° For every j$\in$ A, assign new permanent label

$$d_-(j) = \min\{ d_-(j), d_+(i) + c_{ij} \}$$

4° In this step only one node j*, from all nodes on the tree that have temporary label, gets a permanent label, and that's the one for who follows

$$d_-(j_*) = \min\{ d_-(j) \}$$

hence

$$d_+(j_*) = d_-(j_*).$$

5° Check is j* =t , which means is the final node marked as "temporarily labeled ".

If it is not, make i = j* and go back to step 2°.

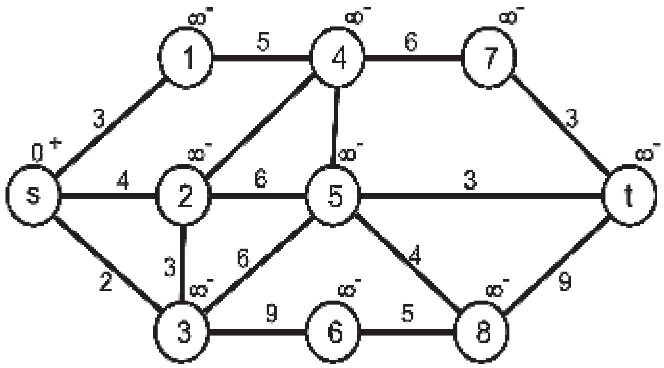
If j* =t it means that the length of a shortest path d+(t) and now the shortest path from s to t has to be reconstructed.

6° The shortest path p = (s, j1, j2, ... , jk, t) is traced backward reconstructing the shortest path from s till t.

Remark The last step of algorithm, the shortest step determination, can be simplified if in step 4°, beside the permanent label, index of node that precede observed node, and based on which it got that label, is remembered.

Example

There is a tree that has to be found a shortest path from node s to node t using the Dijkstra's algorithm.



Picture 3.1 Initial graph

Solution: the tree with the initial labels is on the Picture 3.1. Further goes the iterative part of changing labels:

1) Node 5 follow nodes: l, 2 i 3, As = {l, 2, 3}

d-(1) = min{0+3, ∞} = 3

d-(2) = min{0+4, ∞} = 4

d-(3) = min{0+2, ∞} = 2

min{d-(1), d-(2), d-(3)} = d-(3) = 2 => d+(3) = d-(3)

2) Node 3 follow: 2, 5 i 6, A3 = {2, 5, 6}

d-(2) = min{2+3, 4} = 4

d-(5) = min{2+6, ∞} = 8

d-(6)= min{2+9, ∞} = 11

min{d-(1), d-(2), d-(5), d-(6)} = d-(1) = 3 => d+(1) = d-(1)

3) A1 {4}

d-(4) = min{3+5, ∞} = 5

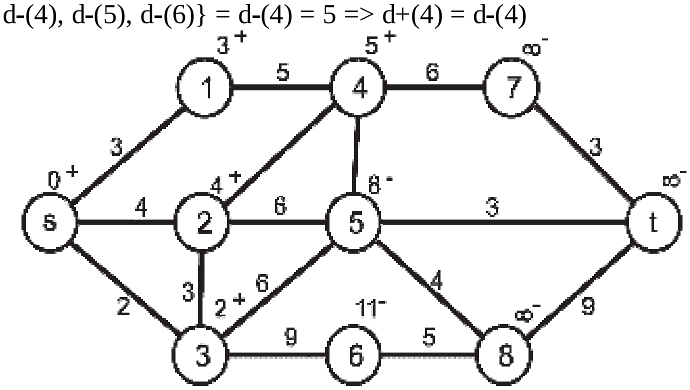
min{d-(2), d-(4), d-(5), d-(6)} = d-(2) = 4 => d+(2) = d-(2)

4) A2 {4,5}

d-(4) = min{3+5, 8} = 5

d-(5) = min{4+6, 8} = 8

min{d-(4), d-(5), d-(6)} = d-(4) = 5 => d+(4) = d-(4)



Picture 3.2 Momentary (instant) position (condition) on the graph in the forth step of the algorithm

The position of labels on the graph, at this moment, can be seen on the Picture 3.2.

5) A4 {5,7}

d-(5) = min{5+2, 8} = 7

d-(7) = min{5+6, ∞} = 11

min{d-(5), d-(6), d-(7)} = d-(5) = 7 => d+(5) = d-(5)

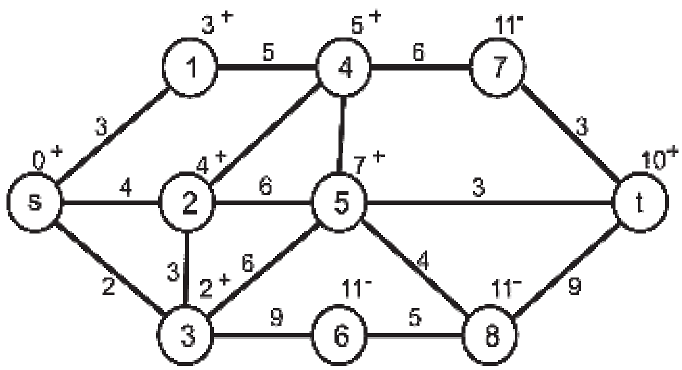
6) A5 {8,t}

d-(8) = min{7+4, ∞} = 11

d-(t) = min{7+3, ∞} = 10

min{d-(6), d-(7), d-(8), d-(t)} = d-(t) = 10 => d+(t) = d-(t)

d+(t) - permanent label => THE END of the iteration procedure . The length of the shortest path from node s to node t is 10. Nodes on the tree are marked as can be seen on picture 3.3.



Picture 3.3 Final solution obtained by using the Dijkstra's algorithm

Let's get on determination of the shortest path.

10- 3 = 7 => node 5,

7-2=5 => node 4,

5-1 = 4 => node 2 and

4-4 = 0 => and we reverted to the root node s .

The shortest path tree of the graph is: (s , 2, 4, 5, t).

It's useful to check does the length of the path that we get in the solution, fits the label d+(t): 3 + 2 + 1 + 4 = 10.

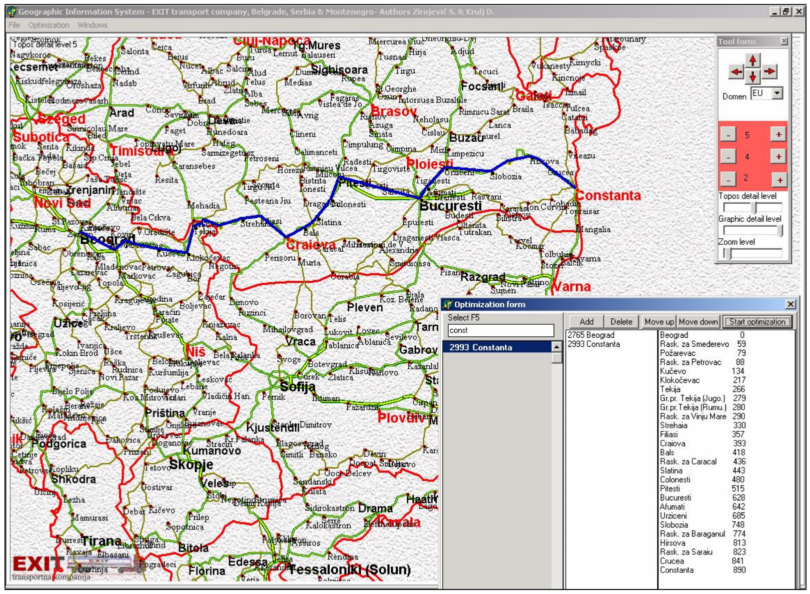
It's important to point out that the permanent label of the node represents the length of the shortest path from node s to the marked node. In solving problems of finding the shortest path between root and all the rest nodes on the tree, it needs to go on with the procedure of labeling nodes until every node get the permanent label.

4. IMPLEMENTATION

Geographical information system (GIS) has been realized in Borland Delphi 6 environment and in data base management system Microsoft SQL Server 2000. Data base contains over ten thousand records that represent populated area and several tens of thousand marks which represent length between the nodes of graph, also and some other objects, roads, borders, rivers, lakes, crossroads, etc.

The process of solving the problem of determining the graphs shortest paths will be represented through several suitable illustrative examples.

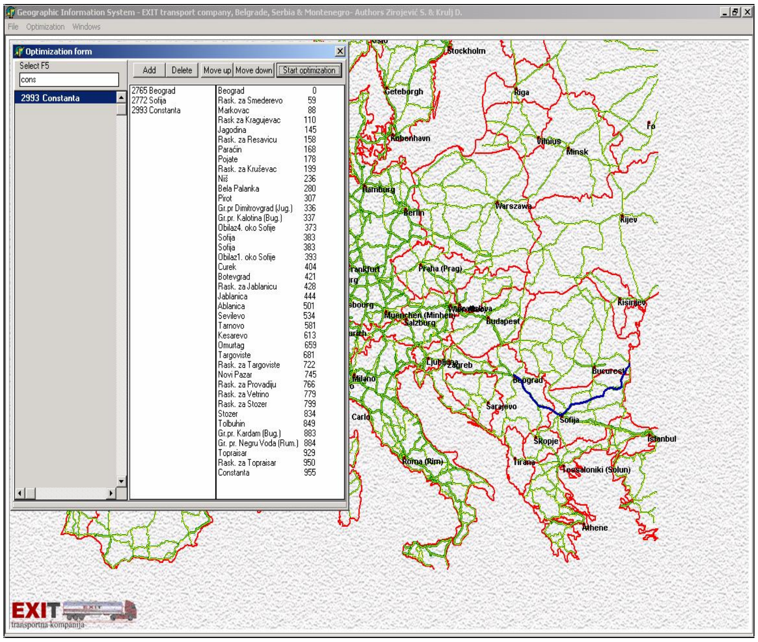
Picture 4.1 shows the shortest path on relation Belgrade-Constanta.



Picture 4.1 Belgrade- Constantia optimal path

As it can bee seen on picture 4.1 right angle shows the list of populated area and their distance from the root area that is passing by on the road from Belgrade to Constantia. Minimum distance Belgrade-Constantia is 890km. Here we must point out that paths must satisfied several criteria, not only the minimum relation, because that particular path might not be optimal by some other criteria especially when we talk about paths via several states (e.g. tax fees, border waiting). Next example of optimization is to find optimal path on relation Belgrade-Constantia complemented with demand that optimal path goes via Sofia. Solution is on the picture 4.2.

As it can bee seen on picture 4.2 minimum distance Belgrade-Constantia via Sofia is 955km.



Picture 4.2 Belgrade- Constantia optimal path via Sofia

The list of all settlements that the path goes by with their cumulative relation can be seen on the left part of picture 4.2.

5. CONCLUSION

The paper shows the problem solving task of finding the graphs shortest paths by using the Dijkstra's algorithm, matched with contemporary methods, instruments and techniques developed within geographical information systems (GIS).

The paper is complemented with presentation of practical implementation of GIS in the process of solving the problem of determining the shortest path tree of the graph. Digital maps, which have been represented in this paper, could be used for many other purposes, like representing demographic data, regional sales revenue data and many other report applications or applications made for arranging geographic resources. Special advantage of this solution, instead of commercial ones, is the possibility to place definition objects on map, and their equal including to the present data, recording to user wishes.

The possibilities of improving represented software solution are various, first of all they relate on placing multi criteria optimization in process of solving problem task of finding the graphs optimal paths. Next important steep in this improving process could be introduction of GPS reference of maps. More about GPS referenced maps designed for you can find in [2 and [8]. GPS referenced map could be used to follow and to differentiate moving objects, like vehicles, trucks and similar kind of transportation. The usage of this kind of map in practice has confirmed as very efficient solution for transport company businesses referring to usage of analog maps or methods based on experience.

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